***Chapter 4 – Induction & Recursion***

***Review questions***

1. Find a recursive definition for an = 1 + (-1)n, n = 0, 1, 2, ...

(i) a0 = 2 and an = an-2, for n > 0

(ii) a0 = 2, a1 = 0 and an = an-2, for n > 1

(iii) a0 = 0, a1 = 2 and an = an-2, for n > 1

1. Find a recursive definition for the set of positive integers NOT divisible by 3

(i) 1 is in S, if x is in S then x + 1 and x + 2 are in S

(ii) 1 is in S, if x is in S then x + 3 is in S

(iii) 1, 2 are in S, if x is in S then x + 3 is in S

1. Study the set S of bit strings defined recursively by:

String 1 belongs to S

If string x belongs to S, then string 11x belongs to S

Which statement is true?

1. 111 11 ∈ S
2. 111111 ∈ S
3. (i) only
4. (ii) only
5. Both
6. None
7. Let f(n) = f(n/3) + 2 and f(1) = 3, where n is divisible by 3.

Find f(27).

1. A recursive definition for the function f(n) = n is:
2. f(1) = 1, f( n ) = n+ f(n-1) for n>1
3. f( n ) = f(n-1) +1 for all n ≥ 1
4. f(1) = 1 and f( n ) = f(n-1) +1 for all n > 1
5. f(1) = 1, f( n ) = n for all n>1
6. None of the others
7. Consider the recursive algorithm:

procedure alg(n : positive integer, a: real number)

if n = 1 then alg(n,a): = a

else alg(n, a) = alg(n-1, a) + a

What is the output if n = 4, a = 2.5?

1. 8
2. 16
3. 10
4. None of these
5. Consider the set A of bit strings deﬁned recursively by

1 ∈ A

if x ∈ A, then x11 ∈ A

Which of the following strings is in A?

1. The empty string λ, the string with no symbols.
2. String 11
3. String 111
4. String 1111
5. To prove the statement " 6 divides n3 - n for all integers n ≥ 0", the mathematical induction method is used as the following:
6. The statement is true for n = 0
7. Suppose \_\_\_\_, the statement is true, that is, "6 divides k3 - k"
8. We have, (k+1)3 - (k+1) = (k3 + 3k2 + 3k + 1) - (k + 1) = k3 - k + 3(k2 + k).

As 6 divides k3 - k (assumption, step 2) and 3(k2 + k) is a multiple of 6, we conclude that (k+1)3 - (k+1) is also a multiple of 6.

1. By induction, 6 divides n3 - n for all integers n ≥ 0.

Fill in the blank at step (2).

a. there exist an integer k ≥ 0

b. for every integer k ≥ 0

c. there are some integers k ≥ 0

9. n is any positive integer, which statements are true?

(i) 12 + 32 + 52 + ... + (2n-1)2 = n3

(ii) 1! + 2! + ... + n! = (n+1)! – 1

1. (i)
2. (ii)
3. None
4. Both

10. Find f (2018) if f (n) = - f(n - 3) and f (0) = 1, f (1) = 4, f(2) = 6.

1. 1
2. 4
3. 6
4. -1
5. -4
6. -6

11. Give a recursive definition of the set A = {…, -7, -4, -1, 2, 5, 8, …}

1. 2 ∈ A; if x ∈ A then x + 3∈ A or x – 3 ∈ A
2. -1∈ A; if x ∈ A then x + 3 ∈ A or x – 3 ∈ A

Which is true?

1. (i)
2. (ii)
3. None
4. Both

